Markscheme

May 2018

Mathematics

Standard level

Paper 1

23 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0**.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r>1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER . . . OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k=3$, but the marks will be for the correct value 3 – there is usually no need for the "$k=$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation *where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".*

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value
the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.

Section A

1. (a) $f(14) = 4$ ***A1*** ***N1***

[1 mark]

(b) attempt to substitute ***(M1)***

eg $g(4)$, $3 \times 4 - 7$

5 ***A1*** ***N2***

[2 marks]

(c) interchanging x and y (seen anywhere) ***(M1)***

eg $x = 3y - 7$

evidence of correct manipulation ***(A1)***

eg $x + 7 = 3y$

$g^{-1}(x) = \frac{x+7}{3}$ ***A1*** ***N3***

[3 marks]

Total [6 marks]

2. (a) recognizing Q_1 or Q_3 (seen anywhere) ***(M1)***

eg $4, 11$, indicated on diagram

$\text{IQR} = 7$ ***A1*** ***N2***

[2 marks]

(b) recognizing the need to find 1.5 IQR ***(M1)***

eg $1.5 \times \text{IQR}$, 1.5×7

valid approach to find k ***(M1)***

eg $10.5 + 11$, $1.5 \times \text{IQR} + Q_3$

21.5 ***(A1)***

$k = 22$ ***A1*** ***N3***

Note: If no working shown, award ***N2*** for an answer of 21.5.

[4 marks]

Total [6 marks]

3. (a) (i) $f(0) = -\frac{1}{2}$ ***A1*** ***N1***

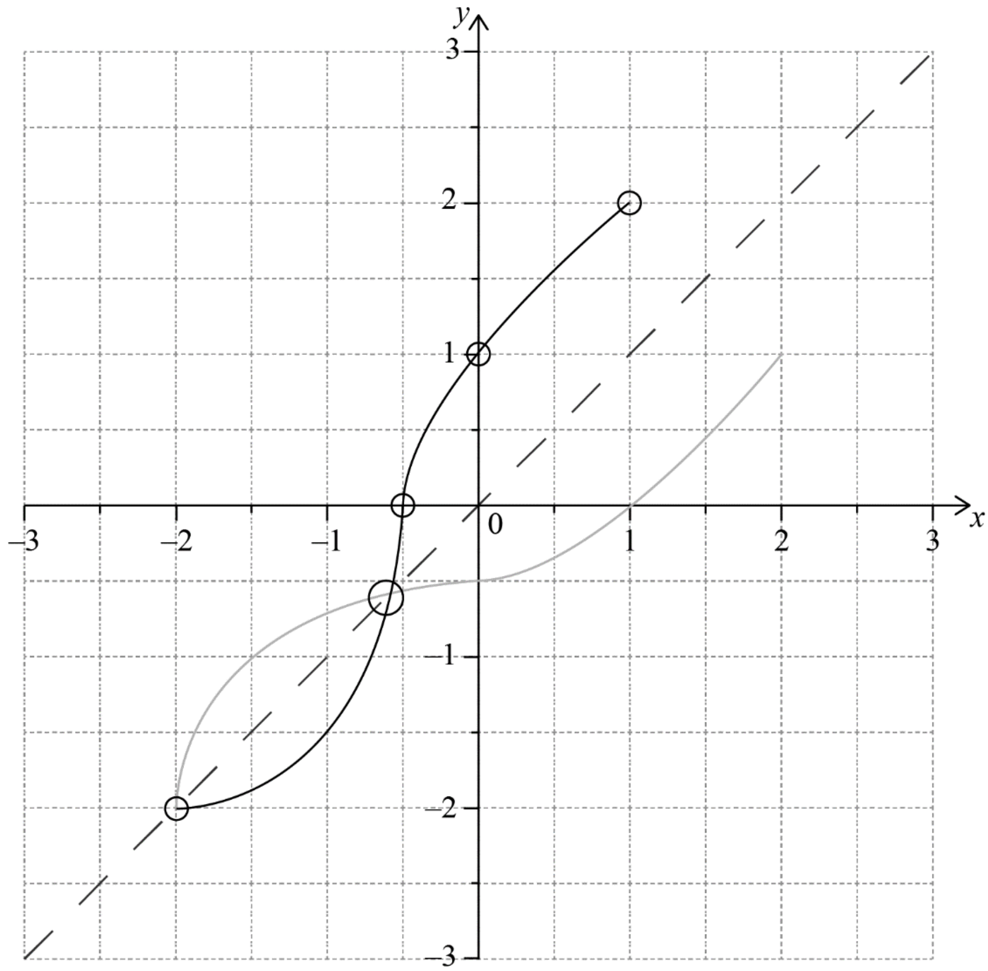
(ii) $f^{-1}(1) = 2$ ***A1*** ***N1***

[2 marks]

(b) $-2 \le y \le 2$, $y \in [-2, 2]$ (accept $-2 \le x \le 2$) ***A1*** ***N1***

[1 mark]

(c)



A1

A1A1A1 ***N4***

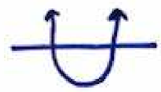
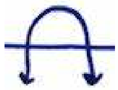
Note:	Award ***A1*** for evidence of approximately correct reflection in $y = x$ with correct curvature. ($y = x$ does not need to be explicitly seen) Only if this mark is awarded, award marks as follows: ***A1*** for both correct invariant points in circles, ***A1*** for the three other points in circles, ***A1*** for correct domain.

[4 marks]

Total [7 marks]

4. (a) **METHOD 1** (using symmetry to find p)

(i) valid approach ***(M1)***

eg $\frac{-1+3}{2}$,  , 

$p = 1$ ***A1*** ***N2***

> **Note:** Award no marks if they work backwards by substituting $a = 2$ into $-\frac{b}{2a}$ to find p.
>
> Do not accept $p = \frac{2}{a}$.

(ii) valid approach ***M1***

eg $-\frac{b}{2a}$, $\frac{4}{2a}$ (might be seen in (i)), $f'(1) = 0$

correct equation ***A1***

eg $\frac{4}{2a} = 1$, $2a(1) - 4 = 0$

$a = 2$ ***AG*** ***N0***

METHOD 2 (calculating a first)

(i) & (ii) valid approach to calculate a ***M1***

eg $a + 4 - c = a(3^2) - 4(3) - c$, $f(-1) = f(3)$

correct working ***A1***

eg $8a = 16$

$a = 2$ ***AG*** ***N0***

valid approach to find p ***(M1)***

eg $-\frac{b}{2a}$, $\frac{4}{2(2)}$

$p = 1$ ***A1*** ***N2***

[4 marks]

(b) valid approach ***(M1)***

eg $f(-1) = 5$, $f(3) = 5$

correct working ***(A1)***

eg $2 + 4 - c = 5$, $18 - 12 - c = 5$

$c = 1$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

5. (a) correct working ***(A1)***

eg $\int \frac{1}{2x-1}\mathrm{d}x$, $\int (2x-1)^{-1}$, $\frac{1}{2x-1}$, $\int \left(\frac{1}{\sqrt{u}}\right)^2 \frac{\mathrm{d}u}{2}$

$\int (f(x))^2\,\mathrm{d}x = \frac{1}{2}\ln(2x-1)+c$ ***A2*** ***N3***

> **Note:** Award ***A1*** for $\frac{1}{2}\ln(2x-1)$.

[3 marks]

(b) attempt to substitute either limits or the function into formula involving f^2 (accept absence of π / $\mathrm{d}x$) ***(M1)***

eg $\int_1^9 y^2\mathrm{d}x$, $\pi\int\left(\frac{1}{\sqrt{2x-1}}\right)^2\mathrm{d}x$, $\left[\frac{1}{2}\ln(2x-1)\right]_1^9$

substituting limits into **their** integral and subtracting (in any order) ***(M1)***

eg $\frac{\pi}{2}(\ln(17)-\ln(1))$, $\pi\left(0-\frac{1}{2}\ln(2\times 9-1)\right)$

correct working involving calculating a log value or using log law ***(A1)***

eg $\ln(1)=0$, $\ln\left(\frac{17}{1}\right)$

$\frac{\pi}{2}\ln 17$ (accept $\pi\ln\sqrt{17}$) ***A1*** ***N3***

> **Note:** Full ***FT*** may be awarded as normal, from their incorrect answer in part (a), however, do not award the final two ***A*** marks unless they involve logarithms.

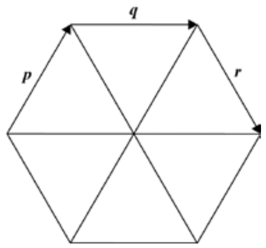
[4 marks]

Total [7 marks]

6. **METHOD 1** (using $|\boldsymbol{p}||2\boldsymbol{q}|\cos\theta$)

finding $\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}$ ***(A1)***

eg $2\boldsymbol{q}$,



$|\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}| = 2\times 3\ \ (=6)$ (seen anywhere) ***A1***

correct angle between $\boldsymbol{p}$ and $\boldsymbol{q}$ (seen anywhere) ***(A1)***

$\frac{\pi}{3}$ (accept 60°)

substitution of **their** values ***(M1)***

eg $3\times 6\times\cos\left(\frac{\pi}{3}\right)$

correct value for $\cos\left(\frac{\pi}{3}\right)$ (seen anywhere) ***(A1)***

eg $\frac{1}{2}$, $3\times 6\times\frac{1}{2}$

$\boldsymbol{p}\bullet(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}) = 9$ ***A1*** ***N3***

METHOD 2 (scalar product using distributive law)

correct expression for scalar distribution ***(A1)***

eg $\boldsymbol{p}\bullet\boldsymbol{p}+\boldsymbol{p}\bullet\boldsymbol{q}+\boldsymbol{p}\bullet\boldsymbol{r}$

three correct angles between the vector pairs (seen anywhere) ***(A2)***

eg 0° between $\boldsymbol{p}$ and $\boldsymbol{p}$, $\frac{\pi}{3}$ between $\boldsymbol{p}$ and $\boldsymbol{q}$, $\frac{2\pi}{3}$ between $\boldsymbol{p}$ and $\boldsymbol{r}$

Note: Award ***A1*** for only two correct angles.

substitution of **their** values ***(M1)***

eg $3.3.\cos 0 + 3.3.\cos\frac{\pi}{3} + 3.3.\cos 120$

one correct value for $\cos 0$, $\cos\left(\frac{\pi}{3}\right)$ or $\cos\left(\frac{2\pi}{3}\right)$ (seen anywhere) ***A1***

eg $\frac{1}{2}$, $3\times 6\times\frac{1}{2}$

$\boldsymbol{p}\bullet(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}) = 9$ ***A1*** ***N3***

continued…

Question 6 continued

METHOD 3 (scalar product using relative position vectors)

valid attempt to find one component of $\boldsymbol{p}$ or $\boldsymbol{r}$ ***(M1)***

eg $\sin 60 = \frac{x}{3}$, $\cos 60 = \frac{x}{3}$, one correct value $\frac{3}{2}, \frac{3\sqrt{3}}{2}, \frac{-3\sqrt{3}}{2}$

one correct vector (two or three dimensions) (seen anywhere) ***A1***

eg $p = \begin{pmatrix} \frac{3}{2} \\ \frac{3\sqrt{3}}{2} \end{pmatrix}$, $q = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$, $r = \begin{pmatrix} \frac{3}{2} \\ -\frac{3\sqrt{3}}{2} \\ 0 \end{pmatrix}$

three correct vectors or $\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r} = 2\boldsymbol{q}$ ***(A1)***

$\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$ (seen anywhere, including scalar product) ***(A1)***

correct working ***(A1)***

eg $\left(\frac{3}{2} \times 6\right) + \left(\frac{3\sqrt{3}}{2} \times 0\right)$, $9+0+0$

$\boldsymbol{p} \bullet (\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}) = 9$ ***A1*** ***N3***

Total [6 marks]

7. recognizing the need to find h' ***(M1)***

recognizing the need to find $h'(3)$ (seen anywhere) ***(M1)***

evidence of choosing chain rule ***(M1)***

eg $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$, $f'\left(g(3)\right) \times g'(3)$, $f'(g) \times g'$

correct working ***(A1)***
eg $f'(7) \times 4$, -5×4

$h'(3) = -20$ ***(A1)***

evidence of taking **their** negative reciprocal for normal ***(M1)***

eg $-\frac{1}{h'(3)}$, $m_1 m_2 = -1$

gradient of normal is $\frac{1}{20}$ ***A1*** ***N4***

Total [7 marks]

Section B

8. (a) evidence of integration *(M1)*

eg $\int f'(x)$

correct integration (accept absence of C) *(A1)(A1)*

eg $x^3 + \frac{18}{2}x^2 + C$, $x^3 + 9x^2$

attempt to substitute $x = -1$ into **their** $f = 0$ (must have C) ***M1***

eg $(-1)^3 + 9(-1)^2 + C = 0$, $-1 + 9 + C = 0$

Note: Award ***M0*** if they substitute into original or differentiated function.

correct working *(A1)*

eg $8 + C = 0$, $C = -8$

$f(x) = x^3 + 9x^2 - 8$ ***A1*** ***N5***

[6 marks]

(b) **METHOD 1** (using 2^{nd} derivative)

recognizing that $f'' = 0$ (seen anywhere) ***M1***

correct expression for f'' *(A1)*

eg $6x + 18$, $6p + 18$

correct working *(A1)*

$6p + 18 = 0$

$p = -3$ ***A1*** ***N3***

METHOD 2 (using 1^{st} derivative)

recognizing the vertex of f' is needed *(M2)*

eg $-\frac{b}{2a}$ (must be clear this is for f')

correct substitution *(A1)*

eg $\frac{-18}{2 \times 3}$

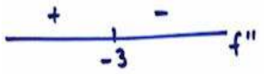
$p = -3$ ***A1*** ***N3***

[4 marks]

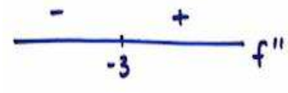
continued…

Question 8 continued

(c) valid attempt to use $f''(x)$ to determine concavity ***(M1)***

eg $f''(x) < 0$, $f''(-2)$, $f''(-4)$, $6x + 18 \leq 0$, 

correct working ***(A1)***

eg $6x + 18 < 0$, $f''(-2) = 6$, $f''(-4) = -6$, 

f concave down for $x < -3$ (do not accept $x \leq -3$) ***A1*** ***N2***

[3 marks]

Total [13 marks]

9. (a) correct approach **A1**

eg $\vec{AO}+\vec{OB}$, $B-A$, $\begin{pmatrix}2\\-4\\-4\end{pmatrix}-\begin{pmatrix}-4\\-12\\1\end{pmatrix}$

$\vec{AB}=\begin{pmatrix}6\\8\\-5\end{pmatrix}$ **AG** **N0**

[1 mark]

(b) (i) any correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (any parameter for t) **A2** **N2**

where $\boldsymbol{a}$ is $\begin{pmatrix}2\\-4\\-4\end{pmatrix}$ or $\begin{pmatrix}-4\\-12\\1\end{pmatrix}$ and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}6\\8\\-5\end{pmatrix}$

eg $\boldsymbol{r}=\begin{pmatrix}-4\\-12\\1\end{pmatrix}+t\begin{pmatrix}6\\8\\-5\end{pmatrix}$, $(x,\ y,\ z)=(2,\ -4,\ -4)+t(6,\ 8,\ -5)$, $\boldsymbol{r}=\begin{pmatrix}-4+6t\\-12+8t\\1-5t\end{pmatrix}$

> **Note:** Award ***A1*** for the form $\boldsymbol{a}+t\boldsymbol{b}$, ***A1*** for the form $\boldsymbol{L}=\boldsymbol{a}+t\boldsymbol{b}$, ***A0*** for the form $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{a}$.

(ii) **METHOD 1** (solving for t)

valid approach **(M1)**

eg $\begin{pmatrix}k\\12\\-k\end{pmatrix}=\begin{pmatrix}2\\-4\\-4\end{pmatrix}+t\begin{pmatrix}6\\8\\-5\end{pmatrix}$, $\begin{pmatrix}k\\12\\-k\end{pmatrix}=\begin{pmatrix}-4\\-12\\1\end{pmatrix}+t\begin{pmatrix}6\\8\\-5\end{pmatrix}$

one correct equation **A1**
eg $-4+8t=12$, $-12+8t=12$

correct value for t **(A1)**
eg $t=2$ or 3

correct substitution **A1**
eg $2+6(2)$, $-4+6(3)$, $-[1+3(-5)]$

$k=14$ **AG** **N0**

continued…

Question 9 continued

METHOD 2 (solving simultaneously)

valid approach **(M1)**

eg $\begin{pmatrix} k \\ 12 \\ -k \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$, $\begin{pmatrix} k \\ 12 \\ -k \end{pmatrix} = \begin{pmatrix} -4 \\ -12 \\ 1 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$

two correct equations in **A1**

eg $k = -4 + 6t,\ -k = 1 - 5t$

EITHER (eliminating k)

correct value for t **(A1)**

eg $t = 2 \text{ or } 3$

correct substitution **A1**

eg $2 + 6(2),\ -4 + 6(3)$

OR (eliminating t)

correct equation(s) **(A1)**

eg $5k + 20 = 30t$ **and** $-6k - 6 = -30t$, $-k = 1 - 5\left(\frac{k+4}{6}\right)$

correct working clearly leading to $k = 14$ **A1**

eg $-k + 14 = 0$, $-6k = 6 - 5k - 20$, $5k = -20 + 6(1 + k)$

THEN

$k = 14$ **AG** **N0**

[6 marks]

continued…

Question 9 continued

(c) (i) correct substitution into scalar product **A1**
eg $(2)(6)-(4)(8)-(4)(-5)$, $12-32+20$

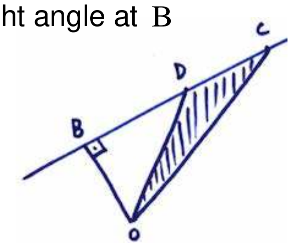
$\vec{OB} \bullet \vec{AB} = 0$ **A1** **N0**

(ii) $\text{O}\hat{\text{B}}\text{A} = \frac{\pi}{2},\ 90^\circ$ $\left(\text{accept } \frac{3\pi}{2},\ 270^\circ\right)$ **A1** **N1**

[3 marks]

(d) **METHOD 1** ($\frac{1}{2} \times \text{height} \times \text{CD}$)

recognizing that OB is altitude of triangle with base CD (seen anywhere) **M1**
eg $\frac{1}{2} \times \left|\vec{OB}\right| \times \left|\vec{CD}\right|$, $\text{OB} \perp \text{CD}$, sketch showing right angle at B



$\vec{CD} = \begin{pmatrix} -6 \\ -8 \\ 5 \end{pmatrix}$ or $\vec{DC} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$ (seen anywhere) ***(A1)***

correct magnitudes (seen anywhere) ***(A1)(A1)***

$\left|\vec{OB}\right| = \sqrt{(2)^2 + (-4)^2 + (-4)^2}\ \left(= \sqrt{36}\right)$

.$\left|\vec{CD}\right| = \sqrt{(-6)^2 + (-8)^2 + (5)^2}\ \left(= \sqrt{125}\right)$

correct substitution into $\frac{1}{2}bh$ **A1**

eg $\frac{1}{2} \times 6 \times \sqrt{125}$

area $= 3\sqrt{125}$, $15\sqrt{5}$ **A1** **N3**

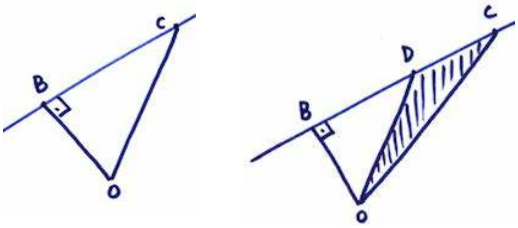
continued…

Question 9 continued

METHOD 2 (subtracting triangles)

recognizing that OB is altitude of either ΔOBD or ΔOBC (seen anywhere) ***M1***

eg $\frac{1}{2}\times\left|\vec{OB}\right|\times\left|\vec{BD}\right|$, $OB \perp BC$, sketch of triangle showing right angle at B



one correct vector $\vec{BD}$ or $\vec{DB}$ or $\vec{BC}$ or $\vec{CB}$ (seen anywhere) ***(A1)***

eg $\vec{BD}=\begin{pmatrix}6\\8\\-5\end{pmatrix}$, $\vec{CB}=\begin{pmatrix}-12\\-16\\10\end{pmatrix}$

$\left|\vec{OB}\right|=\sqrt{(2)^2+(-4)^2+(-4)^2}\left(=\sqrt{36}\right)$ (seen anywhere) ***(A1)***

one correct magnitude of a base (seen anywhere) ***(A1)***

$\left|\vec{BD}\right|=\sqrt{(6)^2+(8)^2+(5)^2}\left(=\sqrt{125}\right)$, $\left|\vec{BC}\right|=\sqrt{144+256+100}\left(=\sqrt{500}\right)$

correct working ***A1***

eg $\frac{1}{2}\times 6\times\sqrt{500}-\frac{1}{2}\times 6\times 5\sqrt{5}$, $\frac{1}{2}\times 6\times\sqrt{500}\times\sin 90-\frac{1}{2}\times 6\times 5\sqrt{5}\times\sin 90$

area $=3\sqrt{125}$, $15\sqrt{5}$ ***A1*** ***N3***

continued…

Question 9 continued

METHOD 3 (using $\frac{1}{2}ab\sin C$ with ΔOCD)

two correct side lengths (seen anywhere) ***(A1)(A1)***

$$\left|\vec{\text{OD}}\right| = \sqrt{(8)^2 + (4)^2 + (-9)^2}\left(= \sqrt{161}\right),\ \left|\vec{\text{CD}}\right| = \sqrt{(-6)^2 + (-8)^2 + (5)^2}\left(= \sqrt{125}\right),$$

$$\left|\vec{\text{OC}}\right| = \sqrt{(14)^2 + (12)^2 + (-14)^2}\left(= \sqrt{536}\right)$$

attempt to find cosine ratio (seen anywhere) ***M1***

eg $\frac{536 - 286}{-2\sqrt{161}\sqrt{125}}$, $\frac{\text{OD} \bullet \text{DC}}{|\text{OD}||\text{DC}|}$

correct working for sine ratio ***A1***

eg $\frac{(125)^2}{161 \times 125} + \sin^2 D = 1$

correct substitution into $\frac{1}{2}ab\sin C$ ***A1***

eg $0.5 \times \sqrt{161} \times \sqrt{125} \times \frac{6}{\sqrt{161}}$

area $= 3\sqrt{125}$, $15\sqrt{5}$ ***A1*** ***N3***

[6 marks]

Total [16 marks]

10. (a) (i) valid approach ***(M1)***

eg $\frac{u_2}{u_1}, \frac{u_1}{u_2}$

$r = \frac{12\sin^2\theta}{18} \left(= \frac{2\sin^2\theta}{3}\right)$ ***A1*** ***N2***

(ii) recognizing that $\sin\theta$ is bounded ***(M1)***

eg $0 \le \sin^2\theta \le 1$, $-1 \le \sin\theta \le 1$, $-1 < \sin\theta < 1$

$0 < r \le \frac{2}{3}$ ***A2*** ***N3***

Note: If working shown, award ***M1A1*** for correct values with incorrect inequality sign(s).
If no working shown, award ***N1*** for correct values with incorrect inequality sign(s).

[5 marks]

(b) correct substitution into formula for infinite sum ***A1***

eg $\frac{18}{1 - \frac{2\sin^2\theta}{3}}$

evidence of choosing an appropriate rule for $\cos 2\theta$ (seen anywhere) ***(M1)***

eg $\cos 2\theta = 1 - 2\sin^2\theta$

correct substitution of identity/working (seen anywhere) ***(A1)***

eg $\frac{18}{1 - \frac{2}{3}\left(\frac{1 - \cos 2\theta}{2}\right)}$, $\frac{54}{3 - 2\left(\frac{1 - \cos 2\theta}{2}\right)}$, $\frac{18}{\frac{3 - 2\sin^2\theta}{3}}$

correct working that clearly leads to the given answer ***A1***

eg $\frac{18 \times 3}{2 + (1 - 2\sin^2\theta)}$, $\frac{54}{3 - (1 - \cos 2\theta)}$

$\frac{54}{2 + \cos(2\theta)}$ ***AG*** ***N0***

[4 marks]

continued…

Question 10 continued

(c) **METHOD 1** (using differentiation)

recognizing $\frac{\mathrm{d}S_\infty}{\mathrm{d}\theta} = 0$ (seen anywhere) ***(M1)***

finding any correct expression for $\frac{\mathrm{d}S_\infty}{\mathrm{d}\theta}$ ***(A1)***

eg $\frac{0 - 54 \times (-2\sin 2\theta)}{(2 + \cos 2\theta)^2}$, $-54(2 + \cos 2\theta)^{-2}(-2\sin 2\theta)$

correct working ***(A1)***
eg $\sin 2\theta = 0$

any correct value for $\sin^{-1}(0)$ (seen anywhere) ***(A1)***
eg $0,\ \pi,\ \ldots$, sketch of sine curve with x-intercept(s) marked

both correct values for 2θ (ignore additional values) ***(A1)***
$2\theta = \pi,\ 3\pi$ (accept values in degrees)

both correct answers $\theta = \frac{\pi}{2},\ \frac{3\pi}{2}$ ***A1*** ***N4***

Note:	Award ***A0*** if either or both correct answers are given in degrees. Award ***A0*** if additional values are given.

continued…

Question 10 continued

METHOD 2 (using denominator)

recognizing when S_∞ is greatest ***(M1)***

eg $2+\cos 2\theta$ is a minimum, $1-r$ is smallest

correct working ***(A1)***

eg minimum value of $2+\cos 2\theta$ is 1, minimum $r=\frac{2}{3}$

correct working ***(A1)***

eg $\cos 2\theta=-1$, $\frac{2}{3}\sin^2\theta=\frac{2}{3}$, $\sin^2\theta=1$

EITHER (using $\cos 2\theta$)

any correct value for $\cos^{-1}(-1)$ (seen anywhere) ***(A1)***

eg π, 3π, ... (accept values in degrees), sketch of cosine curve with x-intercept(s) marked

both correct values for 2θ (ignore additional values) ***(A1)***

$2\theta=\pi$, 3π (accept values in degrees)

OR (using $\sin\theta$**)**

$\sin\theta=\pm 1$ ***(A1)***

$\sin^{-1}(1)=\frac{\pi}{2}$ (accept values in degrees) (seen anywhere) ***A1***

THEN

both correct answers $\theta=\frac{\pi}{2}, \frac{3\pi}{2}$ ***A1*** ***N4***

Note:	Award ***A0*** if either or both correct answers are given in degrees. Award ***A0*** if additional values are given.

[6 marks]

Total [15 marks]